

AN UNITED METHOD CALCULATING NEUTRON FLUENCE ATTENUATION AND GAMMA-RAY  
SELF-ABSORPTION IN A LARGE CYLINDRICAL SAMPLE FOR  $(n,x\gamma)$  EXPERIMENT\*

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**Abstract:** An united formula calculating neutron fluence attenuation and gamma-ray self-absorption in a large cylindrical sample was proposed, which can be used in measurements of gamma-ray production cross sections from fast neutron induced  $(n,x\gamma)$  reactions. The position correlation of these two effects was taken into account in the deduction. The formula has obvious physical meaning. The preliminary calculations were completed at a IBM-PC microcomputer, the comparisons with some other methods were made, and satisfactory results were obtained.

(nuclear reaction, cylindrical sample, neutron fluence attenuation, gamma-ray self-absorption, gamma-ray production cross section)

### Introduction

Measurements of associated gamma-ray cross sections from various nuclides under fast neutron bombardment are of interest for theory researches and practical applications. In the measurements by means of pulsed neutron source large cylindrical samples are usually adopted. There are attenuation of incident neutron and absorption of outgoing gamma-ray by the sample, and amount of attenuation and absorption must be determined to obtain correct absolute gamma-ray production cross sections from the neutron fluence incident upon the sample and gamma-ray yields determined by the experiment. Certainly, except above two corrections, other corrections, for example, neutron multiple scattering correction, neutron attenuation correction in the backing and cooling water of the neutron target, the correction for contributions from activated gamma rays and so on, must be also considered. But the experience has proved that all other corrections aren't usually larger than 15% and corrections of neutron fluence attenuation and gamma-ray self-absorption are usually of amount from tens percents to several hundred percents<sup>1</sup>. For example, for Fe sample in 3cm diameter and 3cm high the attenuation amount of 14.9MeV neutron is 25%, and self-absorption amount of 847keV gamma-ray from this sample under 14.9MeV neutron bombardment is 60% or so, but all of other corrections are only about 12%. Thus it

can be seen that it is of interest for measurements of discrete gamma-ray production cross sections from the reactions induced by fast neutrons to search a reliable and fast method calculating two corrections mentioned above.

In previous experiments neutron fluence attenuation correction in a large cylindrical sample was usually calculated by means of Monte-Carlo method proposed by J.B. Parker et al.<sup>2</sup> or a semi-empirical analytic method proposed by C.A. Engebrecht<sup>3</sup>. Although reliability of the former is good, its calculation is time-consuming, and calculation of the later is simple, but its reliability poorer. Gamma-ray self-absorption correction was usually calculated by an integral formula proposed by Boring(1960) or an approximated analytic formula proposed by J.K. Dickens<sup>4</sup>, but two methods consider gamma-ray self-absorption correction independent of processes of neutron fluence attenuation. This consideration doesn't agree with actual status. In fact, neutron fluence density in the sample isn't homogeneous because of the following factors: 1. the neutron fluence varies as diametral direction for a point neutron source, and 2. neutron fluence is attenuated by the sample material. Thus the contributions from different position of the sample to a gamma-ray yield corresponding to certain energy, which was measured by the gamma-ray detector at certain angle relative to incident neutron direction, aren't different, that is, gamma-ray

self-absorption correction is a function of position in sample, and depends on neutron attenuation status in that point directly. In Boring method only factor 1 was considered. In J.K. Dickens method all of the two factors hadn't been considered. Perhaps this is important one of reasons inducing discrepancies among previous experimental data of gamma-ray production cross sections obtained by different laboratories.

In present paper an united formula calculating two corrections mentioned above was deduced from actual transportation processes of neutrons and gamma-rays in large cylindrical sample. The two factors mentioned above were strictly considered in the deduction.

#### Deduction of United Calculation Formula

In the measurements of discrete gamma-ray production cross sections from fast neutron induced reactions general geometry arrangements are shown in Fig.1 and Fig.2. It is assumed that neutron source is a point one. Sample is a cylinder, radius and height of which are RC cm and H cm, molecular weight of sample element A, weight of sample W g. Cylinder axis of sample is perpendicular to the plane defined by lines from the center of sample to the center of the neutron target and the center of gamma-ray detector, Distance from neutron target to the sample is D. Neutron fluence was measured by means of associated particle method, and gamma-ray spectrum was measured by Ge(Li) detector. A beam of monoenergetic neutrons with energy  $E_n$  from neutron source S goes into sample through A and reaches B with definite probability, and interacts with the nuclei in differential volume element (equal to  $RdRd\theta$ ) near point B. If neutron fluence per steradian in sample direction is  $\phi_n$ , and differential cross section of gamma-ray with energy  $E_\gamma$  from the reaction is  $\sigma(E_\gamma, \theta)$  it can be known from definition of differential cross section that emittance rate of gamma-ray with energy  $E_\gamma$  in steradian of  $\theta$  direction is following

$$dn_{\gamma_0}(E_\gamma, \theta) = \frac{\rho \phi_n N_A \sigma(E_\gamma, \theta) \exp(-\mu_n(E_n) \overline{AB})}{AR} dRd\theta dh \quad (1)$$

in which  $R = \overline{SB}$ ,  $N_A$  is Avogadro constant,  $\mu_n(E_n)$  macro absorption cross section of sample nuclide to neutron with energy  $E_n$ ,  $\rho$  density of sample.

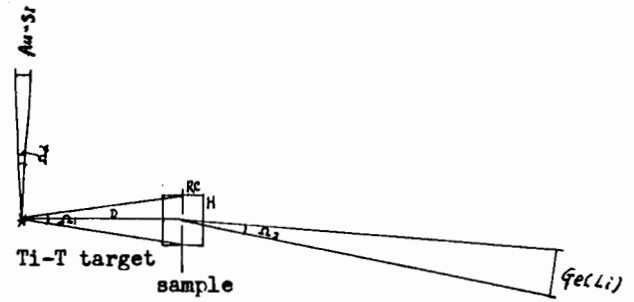


Fig.1 The geometry arrangement in the experiment

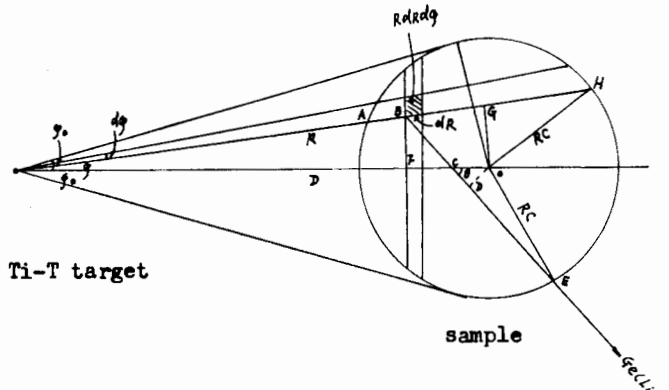


Fig.2 The geometry used in deducting the formulae calculating neutron and gamma ray attenuation in sample

If density of sample is homogeneous,  $\rho$  can be written as

$$\rho = \frac{W}{\pi RC^2 H} \quad (2)$$

insert (2) into (1) we have

$$dn_{\gamma_0}(E_\gamma, \theta) = \frac{W \phi_n N_A \sigma(E_\gamma, \theta) \exp(-\mu_n(E_n) \overline{AB})}{\pi RC^2 H A R} dRd\theta dh \quad (3)$$

If a part of gamma-rays with energy  $E_\gamma$  produced in point B exit from point E on surface of sample along  $\theta$  direction and reach Ge(Li) detector, count rate of gamma-ray with energy  $E_\gamma$  from sample in Ge(Li) detector is  $n_\gamma(E_\gamma, \theta)$ , solid angle subtended by Ge(Li) detector to sample is  $\Omega_2$ , intrinsic and geometrical efficiencies of Ge(Li) detector are  $\xi_b(E_\gamma)$  and  $\xi(E_\gamma)$  respectively we have

$$\frac{dV}{V} n_\gamma(E_\gamma, \theta) = \Omega_2 \exp(-\mu_\gamma(E_\gamma) \overline{BE}) \xi_b(E_\gamma) dn_{\gamma_0}(E_\gamma, \theta) = 4\pi \xi(E_\gamma) \exp(-\mu_\gamma(E_\gamma) \overline{BE}) dn_{\gamma_0}(E_\gamma, \theta) \quad (4)$$

insert (3) to (4) we have

$$\frac{dV}{V} n_{\gamma}(E_{\gamma}, \theta) = \frac{4\xi(E_{\gamma})W\phi\Omega_1 N_A \sigma(E_{\gamma}, \theta)}{\overline{RC}^2 \text{HAR}} \cdot \exp(-\mu_n(E_n)\overline{AB}) \exp(-\mu_{\gamma}(E_{\gamma})\overline{BE}) dR d\phi dh \quad (5)$$

The neutron fluence rate incident upon the sample determined by associated particle method is following

$$\phi_{\Omega 1} = \frac{n_{\alpha}}{\Omega_{\alpha}} A_{\alpha}(\phi) \quad (6)$$

in which  $n_{\alpha}$  is count rate of alpha particles,  $\Omega_{\alpha}$  is the solid angle subtended by the alpha particle detector with respect to the neutron target,  $A_{\alpha}(\phi)$  an anisotropy factor of neutron emittance from  $T(d,n)^4\text{He}$  reactions. Inserting (6) to (5) and integrating (5) over the sample volume  $V$  we have

$$n_{\gamma}(E_{\gamma}, \theta) = \frac{4\xi(E_{\gamma})Wn_{\alpha}\sigma(E_{\gamma}, \theta)N_A}{\overline{RC}^2 \text{HAR}\Omega_{\alpha}} \int_{-H/2}^{H/2} \int_{-\phi_0}^{\phi_0} \frac{\overline{SG} + \overline{AG}}{\overline{SG} - \overline{AG}} \cdot \frac{A_{\alpha}(\phi) \exp(-\mu_n(E_n)\overline{AB}) \exp(-\mu_{\gamma}(E_{\gamma})\overline{BE})}{R} dR d\phi dh \quad (7)$$

Assume that neutron source  $S$ ,  $R$ ,  $\overline{AB}$ ,  $\overline{BE}$  are on the same plane with center cross section over the sample and  $D$  is much larger than  $RC$ , then  $R$ ,  $\overline{AB}$  and  $\overline{BE}$  are independent of sample height  $h$ . In addition,  $A_{\alpha}(\phi)$  can be approximated as a constant  $A_{\alpha}$  in the solid angle subtended by the sample with respect to the neutron source (the error induced by this proximity is usually less than 0.1%). Thus formula (7) can be changed into following form

$$n_{\gamma}(E_{\gamma}, \theta) = \frac{4\xi(E_{\gamma})Wn_{\alpha}A_{\alpha}N_A\sigma(E_{\gamma}, \theta)}{\overline{RC}^2 A \Omega_{\alpha}} \int_{-\phi_0}^{\phi_0} \frac{\overline{SG} + \overline{AG}}{\overline{SG} - \overline{AG}} \frac{\exp(-\mu_n(E_n)\overline{AB}) \exp(-\mu_{\gamma}(E_{\gamma})\overline{BE})}{R} dR d\phi \quad (8)$$

let

$$f = \frac{1}{\int_{-\phi_0}^{\phi_0} \frac{\overline{SG} + \overline{AG}}{\overline{SG} - \overline{AG}} \frac{\exp(-\mu_n(E_n)\overline{AB}) \exp(-\mu_{\gamma}(E_{\gamma})\overline{BE})}{R} dR d\phi} \quad (9)$$

from (8) and (9) the formula calculating differential cross section can be obtained

$$\sigma(E_{\gamma}, \theta) = \frac{A \Omega_{\alpha} \overline{RC}^2 n_{\gamma}(E_{\gamma}, \theta)}{4Wn_{\alpha}A_{\alpha}N_A} f \quad (10)$$

It is obvious that neutron fluence attenuation

and gamma-ray self-absorption corrections are included in factor  $f$ , in which  $\overline{AB}$ ,  $\overline{BE}$ ,  $\phi_0$ ,  $\overline{SG}$  and  $\overline{AG}$  can be obtained from Fig.2

$$\overline{AB} = R - D \cos \phi + \sqrt{\overline{RC}^2 - D^2 \sin^2 \phi} \quad (11)$$

$$\overline{BE} = R \sin \phi / \sin \theta + (D - R \cos \phi - R \sin \phi \cot \theta) \cos \theta + \sqrt{\overline{RC}^2 - (D - R \cos \phi - R \sin \phi \cot \theta)^2 \sin^2 \theta} \quad (12)$$

$$\phi_0 = \arcsin(\overline{RC}/D) \quad (13)$$

$$\overline{SG} = D \cos \phi \quad (14)$$

$$\overline{AG} = \sqrt{\overline{RC}^2 - D^2 \sin^2 \phi} \quad (15)$$

insert (11), (12), (13), (14) and (15) to (9) we have

$$f = \frac{1}{\int_{-\arcsin(\overline{RC}/D)}^{\arcsin(\overline{RC}/D)} \frac{D \cos \phi + \sqrt{\overline{RC}^2 - D^2 \sin^2 \phi}}{D \cos \phi - \sqrt{\overline{RC}^2 - D^2 \sin^2 \phi}} F(R, \phi, \theta) dR d\phi} \quad (16)$$

in which

$$F(R, \phi, \theta) = \exp\left\{-\mu_n(E_n)(R - D \cos \phi + \sqrt{\overline{RC}^2 - D^2 \sin^2 \phi}) - \mu_{\gamma}(E_{\gamma})\left[\frac{R \sin \phi}{\sin \theta} + (D - R \cos \phi - R \sin \phi \cot \theta) \cos \theta + \sqrt{\overline{RC}^2 - (D - R \cos \phi - R \sin \phi \cot \theta)^2 \sin^2 \theta}\right]/R\right\} \quad (17)$$

(16) is just united calculation formula used in neutron fluence attenuation and gamma-ray self-absorption corrections for  $(n, \gamma)$  experiments. It can be seen from formula (11), (12), (16) and (17) that there is really the position correlation between the two corrections because  $AB$  and  $BE$  are the paths through which incident neutrons and outgoing gamma-rays travel in the sample and are functions of  $R$ .

Only total correction value of the two effects can be given by (16). Sometimes every one of the two corrections must be calculated separately, for example, the comparisons with other methods need to be made. Thus the representations of every correction factors must be found. On the basis of analysis from actual physical process it can be known that because amounts of neutron attenuation in the sample are independent of gamma-ray self-absorption the representation of its correction factor can be obtained by deleting factor  $\exp(-\mu_{\gamma}(E_{\gamma})\overline{BE})$  from (9) and removing effect of neutron fluence density's varying as diameter direction from  $f$  after  $f$  was calculated.

When calculating gamma-ray self-absorption factor  $\exp(-n(E_n)AB)$  can not be deleted from  $f$  because of existence of position correlation between it and neutron fluence attenuation. The representation of calculating gamma-ray self-absorption correction may be obtained by removing effects of neutron fluence attenuation and neutron fluence density's varying as diametral direction from  $f$  at the same time after  $f$  was calculated. In order to reduce writing let

$$\int dRd\phi = \int_{-\arcsin(\overline{RC}/D)}^{\arcsin(\overline{RC}/D)} \int_{D\cos\phi - \sqrt{\overline{RC}^2 - D^2 \sin^2\phi}}^{D\cos\phi + \sqrt{\overline{RC}^2 - D^2 \sin^2\phi}} dRd\phi \quad (18)$$

$$F_R(R) = 1/R \quad (19)$$

$$F_n(R, \phi) = \exp(-\mu_n(E_n)(R - D\cos\phi + \sqrt{\overline{RC}^2 - D^2 \sin^2\phi}))/R \quad (20)$$

If  $f_n$  and  $f_y$  are used for representing neutron fluence attenuation and gamma-ray self-absorption correction factors respectively, from above analysis we have

$$f_n = \int dRd\phi F_R(R) / \int dRd\phi F_n(R, \phi) \quad (21)$$

$$f_y = \int dRd\phi F_n(R, \phi) / \int dRd\phi F(R, \phi, \theta) \quad (22)$$

In the deduction of formula (7) it is assumed that outgoing gamma-rays from every part of the sample to the gamma-ray detector have same outgoing angle  $\theta$ . This assumption is reasonable because diameter or height of the sample is usually much less than the distance from the sample to the gamma-ray detector.

In addition, it must be explained that formula (10) only includes two corrections of neutron fluence attenuation and gamma-ray self-absorption in the sample. In practical application other corrections must be also considered so that correct differential cross sections are obtained.

#### Calculation Results and Analysis

The calculation program of relevant formulae mentioned above was written by means of FORTRAN-77 in a IBM-PC microcomputer. Validity of these formulae was tested by some calculations.

It can be used for testing validity of correction methods that samples with different size are measured under same experimental conditions

because the two corrections depend on size of the sample and reaction cross sections are independent of size of the sample. We have measured differential production cross sections of 847keV gamma-ray from two Fe samples with different size under 14.9MeV neutron bombardment at 90 degree. Differential cross sections were calculated by formula (10), and other corrections were considered at the same time, the contributions of which were not more than 12%. Neutron fluence attenuation correction  $f_n$  and gamma-ray self-absorption correction  $f_y$  were calculated by (21) and (22) respectively. Relative data and results were listed in Table 1.

Table 1. Comparison of experimental results from two Fe samples

sample	$\overline{RC}$ (cm)	H(cm)	$f_n$	$f_y$	$\sigma(847\text{keV}, 90^\circ)$
large one	1.396	2.98	1.253	1.60	67.2 $\pm$ 4.0mb/sr
small one	1.000	2.044	1.180	1.42	67.1 $\pm$ 4.2mb/sr

It is seen from Table 1 that although there are about 1/3 difference between values of two samples and between values of  $f_n$  and  $f_y$  of two samples their differential cross sections are nearly the same. This result proved that the calculations of  $f$  are basically correct.

In Table 2 the results of 14.9MeV neutron fluence attenuation in V, Fe, Co and Nb samples calculated by formula (21) and Monte Carlo method were listed.

Table 2. Comparison of results of neutron fluence attenuation correction calculated by present method and Monte-Carlo method

sample	$\overline{RC}$ (cm)	H (cm)	D (cm)	fn	
				formula (21)	Monte-Carlo
V	1.50	3.00	13.4	1.212	1.210
Fe	1.40	2.98	13.4	1.253	1.250
Co	1.50	3.04	13.4	1.292	1.294
Nb	1.51	3.04	13.4	1.293	1.292

It can be seen from Table 2 that the results of four samples obtained by two methods are nearly consistent. The calculation time of present method is much less than one of Monte-Carlo method. It takes several seconds for present method and 20 minutes for Monte-Carlo method to complete neutron fluence attenuation correction calculation of one sample at a IBM-PC/XT microcomputer.

In Table 3 self-absorption correction results of 847keV gamma-ray from Fe sample under 14.9MeV neutron bombardment at three angles, which were obtained by formula (22) and Boring method, were listed.

Table 3. Comparison between present method and Boring method calculating gamma-ray self-absorption correction

angle	$f_{\gamma}$	
	formula (22)	Boring method
30°	1.705	1.649
90°	1.600	1.604
140°	1.516	1.568

The results listed in Table 3 can be explained by analysis of practical process. Effects of neutron fluence attenuation in the sample and in the case of point neutron source neutron fluence density's varying as diametral direction result in that the neutron fluence density in half sample cylinder facing to neutron source is higher than one in another half sample cylinder. Thus when measurements in smaller angles were made the area with higher neutron fluence density is farther than one with lower neutron fluence density for the gamma-ray detector, and when measurements in larger angles were made opposite status occur. Therefore self-absorption correction amount of gamma-rays measured at smaller angles must be larger than one of gamma-rays measured at larger angles. At 90° because the distance from the area with higher neutron fluence density to the gamma ray detector is the same as one from the area with lower neutron fluence density to the gamma-ray detector, on an average outgoing gamma-rays from two areas to the gamma-ray detector travel through same path in the sample, therefore amount of gamma-ray self-absorption correction must be independent of neutron attenuation status in the sample. When measurement angle increases or decreases from 90° the effect of neutron attenuation status in the sample to gamma-ray self-absorption correction must be larger and larger. The results in Table 3 are consistent with above analyses. It can be seen from Table 3 that the results at 90° calculated by two methods are the same. At 30° and 140° although the change tendencies of the results calculated by two methods is

same there is difference of about 5%. This difference just reflects effect of neutron fluence attenuation to gamma-ray self-absorption in the sample.

From above analyses we can obtain following conclusions:

1. Dickens method can not be used in data precessing of measurements of differential gamma-ray production cross sections because in which the effect of neutron fluence change in the sample to gamma-ray self-absorption correction hadn't at all been considered.

2. Boring method is suitable only for data precessing of this type of experiment at 90°.

3. Although Monte-Carlo method is correct for neutron fluence attenuation correction it can not be used in data precessing of this type of experiments until position correlation of neutron fluence attenuation and gamma-ray self-absorption corrections is settled.

4. The united calculation formula given in present paper is more suitable for data precessing of (n,xγ) experiments.

#### REFERENCE

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\*The work supported by the National Natural Science Foundation of China and Ministry of Nuclear Industry of China